# Valuing Branded Businesses*

Natalie Mizik
Columbia University
Graduate School of Business
3022 Broadway, Uris Hall, Room 513
New York, NY 10027-6902
e-mail: nm2079@columbia.edu

Robert Jacobson
University of Washington
Foster School of Business
Box 353200
Seattle, WA 98195-3200
e-mail: yusho@u.washington.edu

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# Valuing Branded Businesses


## Abstract

We present an approach for valuing branded businesses that enhances traditional multiplier-based valuation approaches by explicitly incorporating brand characteristics into the model. Empirical analysis indicates that valuation accuracy can be significantly improved by incorporating information about the properties of the firm's brand asset into a valuation framework. We find that brand metrics have statistically significant associations with valuation multipliers and add incremental explanatory power to accounting variables in explaining valuation multipliers. Out-of-sample analysis shows a 16% improvement in the mean absolute error for predictions taking into account brand metrics compared to predictions based on accounting variables alone.

## Introduction

With efficient financial markets, the market value of a stock provides an unbiased estimate of the net present value of a firm's future cash flows. As such, stock market valuation typically provides a reliable measure of a firm's value and can be used as a starting point in, for example, merger and acquisition discussions. At times, however, an estimate of firm value is needed, but financial market data are unavailable. This occurs, for example, when a firm is not publicly traded or when a valuation is needed for a business unit rather than the entire corporate entity. For example, Pfizer sold its consumer goods business units to Johnson & Johnson in 2006. Cadbury announced in 2007 that it would be divesting its North American beverage businesses. The Swedish government in 2008, as part of a drive to privatize six state-owned companies, sold V&S, which is best known for its Absolut Vodka brand. The Swedish government also explored the possibility of splitting V&S into separate brands and selling each one individually.

Placing a value on these entities is complicated by the fact that they are "branded businesses," i.e., the brand asset represents a significant portion of the alienable value. Traditional financial valuation approaches have difficulty in valuing businesses with significant intangible assets (Barth at al. 1998, Lie and Lie 2002). Rarely do these traditional valuation approaches explicitly account for the incremental impact of intangibles such as brand assets. Instead, the contribution of brand assets to firm value tends to be dealt with in an ad hoc or subjective fashion.

In this study we present a multiplier-based valuation approach that explicitly incorporates brand characteristics into the business valuation model. Our objective is to advance business valuation methodology by assessing whether that valuation accuracy can be improved by incorporating brand metrics into the valuation framework. As such, the study has a different objective from a stream of research seeking to isolate and place a value on the brand asset (e.g., Ailawadi et al. 2003, Park and Srinivasan 1994, Simon and Sullivan 1993, Srinivasan et al. 2005, Sriram et al. 2007). Our focus is on using brand information to enhance predictive accuracy in the valuation of a branded business as an entire entity.

We apply our branded business valuation methodology to Young and Rubicam's Brand Asset Valuator (BAV) data. Our empirical analysis indicates that valuation can be significantly improved by incorporating information about the characteristics of the firm's brand asset into a valuation framework. We find that brand metrics have statistically significant associations with valuation multipliers and add incremental explanatory power to accounting variables in explaining valuation multipliers. Out-of-sample analysis shows a 16% improvement in predictive power (as measured by mean absolute prediction error) for predictions taking into account brand metrics, compared to predictions based on accounting variables alone.

The rest of the paper is organized as follows. We first review valuation methodologies. We then discuss why and how brand characteristics can be incorporated into a multiplier-based valuation framework. Next, we present our econometric modeling approach and discuss the data used in the analysis. We then provide empirical results showing the extent to which brand metrics help predict the enterprise value-to-sales multiplier. We conclude with discussion and directions for future study.

## Valuation Approaches

A host of different approaches to business valuation exist. While these frameworks make different assumptions, they share some similarities and can be broadly classified into two general valuation approaches (Damodaran 2002).[1] One framework is based on discounted cash flow (DCF) analysis. This approach is referred to as a "direct valuation approach" and seeks to estimate the intrinsic value of an asset based on its fundamentals. It relies on the net present value rule, where the value of an asset is measured based on discounted expected future cash flows.

[1] An additional approach, "contingent claim" valuation, has also been recently developed. It uses option pricing models to measure the value of assets that share option characteristics. This approach it typically used for valuing traded financial assets and has not received much attention for valuing businesses (see Chen and Zhang (2003) for an exception).

Undertaking a DCF valuation requires obtaining estimates (i.e., projections) of future cash flows and of a discount rate, which depends on the riskiness of the firm. While theoretically appealing, DCF analysis is not easy to implement because of the inherent uncertainty associated with the future. Considerable ambiguity exists in making estimates of future cash flows and discount factors. This approach is particularly difficult to implement for IPOs, young firms, firms in dynamic industries, and for firms with significant intangible assets, such as patents or valuable brands (Kim and Ritter 1999). A great deal of judgment and guesswork is typically involved in coming up with the necessary inputs.[2]

Because of the difficulties in implementing DCF valuation, relative valuation methods are commonly used as an alternative approach or as a complementary tool to DCF valuations. Relative valuation methods (also referred to as "comparable firm valuation" or "peer group valuation" approaches) are based on the premise that similar assets should be priced similarly. As such, the value of an asset can be established based on how similar assets are priced in the market. Under relative valuation, the value of an asset is determined based on the pricing of assets having comparable characteristics.

Unlike the DCF valuation approach, relative valuation bypasses explicit projections and does not attempt to identify the intrinsic value of an asset. Instead, it relies on the market mechanism to reveal the asset's price. The underlying assumption is that, on average, the market correctly prices assets and that the average valuation of assets having similar characteristics can be used to ascertain the value of another asset.

***The Basic Multiplier Valuation Methodology***

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[2] Studies show that DCF valuation is affected by the interests and incentives of the party undertaking the valuation. For example, Gilson at al. (2000) report "strategic distortions" in DCF valuations of bankrupt firms. They find significant relationships between DCF valuation errors and conflicting financial interests of stakeholders in the bankruptcy negotiations. Specifically, they find that DCF valuation errors are systematically related to (i) relative bargaining strength of claimholders (junior versus senior), (ii) existence of outside bids, (iii) management's equity stake, and (iv) management turnover.

Relative business valuation approaches make use of "ratio analysis" or "multiplier analysis." First, a set of similar businesses is identified and their market value is linked to a common standardizing factor or "value driver." For example, firm value is often depicted as a multiple of an accounting financial measure such as book value, earnings, or sales. For example, the value of a firm can be expressed as a function of its sales:

[1] $\text{Firm Value}_{it} = \phi_{it} * \text{Sales}_{it}$,

where $\text{Firm Value}_{it}$ is a measure of the value of firm i at time period t, $\text{Sales}_{it}$ is a measure of firm i revenues in time period t, and $\phi_{it}$ is the sales multiplier for firm i at time t. The multiplier $\phi_{it}$ transforms the accounting sales measure (i.e., the value driver) into firm value.

Once multipliers are obtained for a peer group of firms, the average value of the multiplier can be used to value a non-publicly traded business. For example, in its simplest form the valuation of a business could be determined by multiplying the sales of the business by the value-to-sales ratio of comparable publicly traded firms in the same industry.

As noted by, for example, Liu, Nissim, and Thomas (2007), valuation based on multiples boils down a complex function of discount rates and future cash flows into a simple proportional relation: predicted firm value equals to the level of the value driver for the firm times the corresponding multiplier. Because the multiple is an average or typical ratio of firm value to the value driver for a set of firms having similar characteristics, the key consideration in relative valuation methods is determining what characteristics make assets similar or comparable.[3]

Bhojraj and Lee (2002) argue that the choice of the peer group should be a function of the variables that drive cross-sectional variation in a given valuation multiple. They suggest that "any normative approach to selecting comparable firms should reflect the fundamental concepts that underpin equity valuation" (p. 434) and that industry-based approach with firm-specific adjustments

[3] A limitation of relative valuation methods is that since they rely on market valuation of similar firms, they run the risk that an entire sector maybe under- or over-valued.

is a sensible way to capture these factors. They show that predictive performance of multipliers can be significantly enhanced with more thorough and systematic peer group selection.

*Inter-Industry Effects*

As a starting point, relative valuation analysis is typically undertaken at an industry or business sector level to control for business environment effects and for general future growth prospects. Alford (1992) examined the impact of comparables selection rules (e.g., based on industry, size, and earnings growth) on the valuation accuracy using multiples. He reports reduction in valuation errors when peer group is based on industry affiliation. As industrial sectors are defined more narrowly, i.e., moving to the three-digit SIC code level from the one- or two-digit level, valuation accuracy improves, but no error reduction occurred when going from the three to a four-digit SIC level. He also found no improvement in valuation accuracy from controlling for size and growth in addition to industry membership. He concludes that industry membership is an effective criterion for selecting comparable firms.

*Intra-Industry Effects*

While value multipliers have some stability across firms in the same industry, significant intra-industry differences can and do exist (Kim and Ritter 1999). That is, even among firms within the same industry, firms may differ in attributes that affect valuation and hence yield differing value-to-sales ratios. For example, in 2006 Johnson & Johnson paid $16.6 billion for the consumer unit of Pfizer, which had annual sales at the time of $3.9 billion. This is a value-to-sales multiple of 4.3. Proctor and Gamble's 2005 purchase of Gillette for $57 billion represented a value-to-sales ratio of approximately 5.7. Past research points to differences in profitability as the key factor explaining within-industry differences in multipliers.

*Profit Margin*

A sales multiple measures the value of a business relative to the revenue it generates. While its use offers some advantages over other multiples such as an earnings ratio (e.g., a sales figure is more likely to be available than other accounting metrics and it is less subject to accounting distortions and transitory fluctuations), the sales multiple does not reflect differences in profitability across firms. Indeed, a firm can be generating sales but losing money. A firm needs to be able to generate cash flows over the long term for the business to have value. Differences in business value are related to differences in profit margins. Damodaran (2002) provides a model showing how the sales multiplier is an increasing function of profit margin. That is, firms with higher margins, all else equal, should have higher value-to-sales multiples. He finds empirically for specialty chemical firms that a 1 unit increase in net margin is associated with a 5.71 unit increase in the sales multiplier.

Indeed, a number of studies document the role of incorporating earnings and margins in enhancing accuracy of relative valuation methods. Boatsman and Baskin (1981), for example, report that valuation accuracy increases when comparable firms are selected within the same industry based on similarity of earnings growth. Barth et al. (1998, 1999) also find systematic relationships between multiples and financial performance and report that multiples tend to decrease as firm financial health decreases. More recently, Bajaj at al. (2004) provide further evidence to show that firm profitability has an economically important impact on industry-adjusted multiplier ratios.

*Intangible Assets*

Past research is much less clear on how intangible assets, such as brand, should be incorporated into multiplier-based valuation analysis. Most valuation approaches tend to view intangibles as impacting accounting fundamentals and, as such, already being incorporated into multiplier analysis through their impact on contemporaneous accounting metrics. Damodaran (2002), for example, argues that the impact of intangibles is already reflected in higher profit margins and, as such, should not be treated separately as that would amount to double-counting.

Barth et al. (1998) follow a similar logic in their study. They start with a firm valuation model that defines firm value as an additive function of recognized net assets (book value of equity) and unrecognized net assets (i.e., intangibles). Because unrecognized net assets are not directly observable, they argue that net income is providing information about the unrecognized intangible assets. Consistent with their arguments, they find that net income is more informative in explaining firm value in industries with high levels of unrecognized intangibles than in industries with low levels of intangibles. Their findings suggest that intangible assets (or at least a portion of intangible assets) are reflected in the contemporaneous financial performance of the firm.

However, current-term accounting measures may not fully reflect factors affecting differences in future-term profitability and valuation. Non-financial measures can provide useful incremental information to accounting metrics in determining valuation. For example, firms may have different brand attributes that have long-term profit implications. While some of these brand attribute differences will be reflected in the current-term accounting metrics, some of the effects might be long-term or occur only in the future. As such, they would not be reflected in contemporaneous performance metrics (Mizik and Jacobson, 2008).

Kohlbeck and Warfield (2007) address the role of intangibles in valuing firms in the banking industry. They propose that intangibles have two effects on future cash flow stream. As do Barth et al. (1998), they argue that intangibles lead to greater earnings in the current period. However, they also argue for an additional influence of intangibles that pertains to the dynamic properties of earnings. They suggest and provide evidence consistent with firms with greater intangibles having more persistent earnings. This higher persistence results in a greater net present value of cash flows and, as such, a higher earnings multiplier.

***Incorporating the Impact of Brand Asset into Business Valuation***

To the best of our knowledge, the extent to which intangible assets, such as brands, have an impact on valuation multipliers, incremental to their effects on current accounting performance, has not been directly examined.[4] Marketing theory and the empirical evidence, however, support the

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[4] Kerin and Sethuraman (1998) assess the association between the market-to-book equity ratio and the Financial World estimate of brand value. So, they, as do we, link a brand metric to a stock market multiplier. A central

view that such an impact exists (e.g., Hauser et al. 1994, Srivastava et al. 1998, Dekimpe and Hanssens 1999, Pauwels et al. 2004, Van Heerde et al. 2007). Indeed, marketing literature has highlighted two major ways brands can affect firm financial performance. Brands increase perceived value of the products and therefore attract customers and influence their preference and choice (e.g., Swait and Erdem 2007). This impacts firm current financial outcomes. This immediate brand impact can be reflected in higher price premium, volume premium, and revenue premium of a branded product (Ailawadi et al. 2003, Keller and Lehmann 2003, 2006).

An additional, incremental impact, however, has also been highlighted. Brands build customer loyalty and attachment and therefore can affect future consumption patterns and firm risk (Keller 1993, Johnson et al. 2006, McAlister at al. 2007). This, in turn, would affect firm future financial performance. Several studies in marketing have documented the link between perceptual brand attributes (e.g., Aaker and Jacobson 1994, 2001, Mitra and Golder 2006, Mizik and Jacobson 2008) or product-market-based brand measures (Barth et al. 1998) and firm future-term financial performance. As such, we hypothesize that brand assets will have a direct impact on valuation multipliers, incremental to their effects on current accounting performance.

One of the reasons why intangible assets have not received separate attention and have not been assessed for incremental impact in accounting valuation models is because they are unmeasured in traditional accounting reports. However, with recent efforts to track some marketing-related intangible assets on a consistent basis (e.g., customer satisfaction, perceptual brand assets, customer loyalty, etc.), explicit measures of intangible assets are becoming available. This allows for the incremental contribution of these intangibles to valuation multipliers to be directly modeled and

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difference in the approaches relates to the use of profitability accounting information. We include a measure of profitability (i.e., return on sales) in our model, they do not. As such, in contrast to Kerin and Sethuraman (1998), we assess whether the brand asset has incremental predictive power to earnings information. As noted by Kerin and Sethuraman (1998, p. 273) their study confirms that the Financial World brand value estimate and the market-to-book ratio are jointly correlated with cash flows. Our analysis goes beyond an assessment of joint correlation and investigates the incremental predictive power of brand metrics.

assessed. This is what we undertake. To the extent brand metrics have predictive power, which is incremental to current accounting measures, they can be used to improve estimates of business value.

In order to explicitly allow for the effects of brand assets on firm valuation, we can decompose $\text{Sales}_{it}$ in Equation [1] into two parts: (i) their baseline amount (e.g., the industry average) and (ii) the amount derived from the presence of brand assets that are above or below the baseline. We can decompose the value multiple ($\phi_{it}$) in Equation [1] into three components: (i) the baseline amount (e.g., the industry average), (ii) the amount derived from the above or below the base level of firm financial performance in the industry, and (iii) the amount derived from the presence of brand assets that are above or below the base level (e.g., industry average). This decomposition leads to:

$$[2]\ \text{Firm Value}_{it} = [\phi(\text{Baseline})_{it} + \phi(\text{Current Profitability})_{it} + \phi(\text{Brand})_{it}] * [\text{Sales(Baseline)}_{it} + \text{Sales(Brand)}_{it}] + \varepsilon_{it},$$

where $\text{Sales(Baseline)}_{it}$ is the sales level when the brand assets are at the industry average, $\text{Sales(Brand)}_{it}$ is the amount of incremental sales due to the brand assets of a firm deviating form the industry average brand assets, $\phi(\text{Baseline})_{it}$ is the sales multiplier when firm profitability and the brand assets are at the industry average, $\phi(\text{Current Profitability})_{it}$ is the incremental sales multiplier due to firm profitability deviating form the industry average level, and $\phi(\text{Brand})_{it}$ is the incremental sales multiplier due to the brand assets of a firm deviating from the industry average and not reflected in current profitability.

Equation [2] depicts brand assets impacting firm value in two main ways. They affect both current sales (i.e., firms with better brand assets have higher levels of sales) and the sales multiplier $\phi$. Equation [2] expands recent operationalizations of brand equity by explicitly recognizing potential future-term impact of the brand. For example, the $\text{Sales(Brand)}_{it}$ component in the equation [2] is most similar to and captures the essence of the revenue premium-based metric of brand equity advocated by, for example, Ailawadi at al. (2003).[5] However, Equation [2] also allows for brand effects on future-term financial performance that are not captured in (i.e., are incremental to) the impact on current profitability. As such, Equation [2] allows the differences in brand assets across firms within a sector to be reflected not just in different sales levels or through an indirect effect on the multiplier through brand's impact on current profitability, but also through a direct brand effect on the multiplier.

## Empirical Methodology

In order to undertake a valuation analysis of a business, two inputs are required: a measure of a firm's "value driver" or valuation base (we use sales) and an appropriate multiplier. The measure of the value driver is available from accounting reports or profit/loss statements. As such, it is taken as a given input. It is the determination of the appropriate multiplier that is the focus of valuation analysis. However, to better understand the interplay of brand effects in determining business valuation, in our study we also present analyses assessing the effect of brand dimensions on the sales value driver. We then compare the role of brand characteristics on sales versus the effect of brand characteristics on the sales multiplier.

***Assessing the Impact of Brand Assets on Sales***

Consider a sales model of the form:

$$[3]\ \log(\text{Sales}_{it}) = \lambda_i + \sum_{b=1}^{5} \theta_b \text{*Brand Asset}_{bit} + \sum_{t=1}^{T} \sum_{k=1}^{K} \zeta(k,t)\text{*}S(kt) + \varepsilon_{it},$$

where $\text{Sales}_{it}$ is total revenue of firm i in year t, $\lambda_i$ is a firm specific effect, Brand $\text{Asset}_{bit}$ is a set of brand metrics, and S(kt) is an indicator function that is equal to 1 if the firm is in sector k for period t and 0 otherwise. Past research has documented that sales series typically have a unit root (and we confirm this with formal tests in our data) that is reflected in the error term in Equation [3] being of

[5] One notable difference in the current specification and that of Ailawadi at al. (2003) is that we use the industry average rather than a generic brand as a baseline of comparison. Since our analysis is undertaken across different industries, our choice is motivated by the observation that in some cases generics are not available (e.g., airlines).

the form: $\varepsilon_{it} = \varepsilon_{it-1} + \eta_{it}$, where $\eta_{it}$ is a white noise error. To address this error structure, we take first differences of the data, which removes the fixed effect and unit root structure of the error and yields:

$$[4]\ \Delta\log(\text{Sales}_{it}) = \sum_{b=1}^{5} \theta_b * \Delta\text{Brand Asset}_{bit} + \sum_{t=1}^{T} \sum_{k=1}^{K} \zeta(k,t) * \Delta S(kt) + \eta_{it}.$$

While Equation [4] links sales growth to the change in brand assets, the coefficient $\theta_b$ can be interpreted as depicting the effect of brand asset b on sales in Equation [3] (i.e., the structural interpretation of the model parameters does not change under a first-difference transformation).

***Assessing the Impact of Brand Assets on the Sales Multiplier***

The cornerstone of branded business valuation is not the sales equation, as sales is a known input. Rather, the key is predicting the value for the sales multiplier.

Consider a valuation multiplier model of the form:

$$[5]\ \text{Value-to-Sales}_{it} = \phi_{it} = \alpha_i + \beta * X_{it} + \sum_{t=1}^{T} \sum_{k=1}^{K} \gamma(k,t) * S(kt) + \varepsilon_{it},$$

where Value-to-Sales$_{it}$ is the value-to-sales ratio ($\phi_{it}$) for firm i in time period t, $X_{it}$ is a vector of observed explanatory factors (which might include accounting metrics such as return on sales and non-financial measures such as brand metrics), $\alpha_i$ is a firm-specific constant, S(kt) is an indicator function that takes on the value of 1 if the firm is in sector k for period t; 0 otherwise, and $\varepsilon_{it}$ is a white noise error. Equation 5 depicts the value-to-sales ratio of a firm as a function of a yearly industry mean, a set of observed firm factors $X_{it}$, and a firm specific constant.

While Equation 5 could be estimated for publicly traded firms, it could not be then used to predict the Value-to-Sales ratio for other non-publicly traded firms and business units of publicly traded firms. That is, past measures of the Value-to-Sales ratio, which are needed in order to estimate the firm-specific mean $\alpha_i$, are not available for the non-publicly traded entities. As such, Equation 5 cannot be implemented as a prediction model for private firms or business units.

Given the information available, the question is how best to use this limited information to generate value-to-sales predictions. One approach would be to estimate Equation 5 for publicly traded firms using fixed effects estimation, obtain an estimate of $\hat{\beta}$, and then use this estimate to compute the value-to-sales multiplier of firm j as:

[6] $\hat{\phi}_{jt} = \hat{\beta} * X_{jt} + \hat{\gamma}(k,t) * S(kt)$.

Of note in the Equation 6 prediction model is that, unlike Equation 5, it sets the firm-specific effect $\alpha_i$ to be equal to zero. This constraint, however, may well be inappropriate. In fact, $\alpha_i$ might in fact be a major component determining the value-to-sales multiplier.

An alternative approach would be to utilize an estimation model that does not make this unwarranted assumption and does not require the use of information that is unavailable for prediction purposes. Rather, the model specification can be modified to only utilize information that would be available for prediction purposes. This valuation multiplier model would be of the following form:

[7] $\phi_{it} = \delta * X_{it} + \sum_{t=1}^{T} \sum_{k=1}^{K} \gamma(k,t) * S(kt) + \eta_{it}$.

The estimated coefficients from Equation 7 could then be used to generate the value-to-sales multiplier for firm j as:

[8] $\hat{\phi}_{jt} = \hat{\delta} * X_{jt} + \hat{\gamma}(k,t) * S(kt)$.

Equation 7 explicitly omits the firm-specific effects $\alpha_i$ at the estimation stage. To the extent that the firm-specific effects $\alpha_i$ are correlated with explanatory factors in $X_{it}$, the estimated effects of $\hat{\delta}$ will pick up some of their influence. That is, least squares estimation of Equation 7 will generate $\hat{\delta}$ estimates that compound the effects of $X_{it}$ and $\alpha_i$. In other words, the model parameters estimated in Equation 7 are biased, i.e., $E(\hat{\delta}) \neq \beta$. The bias reflects some of the impact of the unobserved firm-specific information in $\alpha_i$. The magnitude of the bias and the extent to which the unobserved information will be accounted for in the model depends on the classic omitted variable bias conditions, i.e., the corr ($\alpha_i$, $X_{it}$), the magnitude of $\alpha_i$, and the variation of $\alpha_i$ compared to the variation of $X_{it}$.

When the analysis is focused on the causal effect of an explanatory factor, Equation 5 and the fixed effects estimator $\hat{\beta}$ is the more appropriate estimator because it is unbiased and reflects only the impact of factor $X_{it}$ on $\phi_{it}$. In the forecasting context, however, the issue does not revolve around obtaining an unbiased estimate of the structural parameters $\beta$. Rather, the goal is to extract as much information content from the series $X_{it}$ as possible. The estimated parameter $\hat{\delta}$ will be a biased estimate of $\beta$, with the estimate reflecting not just the effect of $X_{it}$, but also some of the impact of firm-specific effect $\alpha_i$. In a forecasting context, this is desirable in that it will improve predictive performance of the model. Instead of seeking to minimize the effect of omitted factors, forecasting is enhanced by incorporating this additional information ("bias") into the estimated effect.[6] As such, it is the biased estimate $\hat{\delta}$ that is more advantageous for use in a forecasting context. We, therefore, make use of Equation 7 for our empirical analysis.

***Hypothesis Testing in the Presence of Correlated Errors***

A difficulty in undertaking analysis based on Equation 7 relates to evaluating the statistical significance of the coefficient estimate $\hat{\delta}$. The problem is that since not all of the impact of $\alpha_i$ will be captured by $X_{it}$, the error in Equation 7 will have a firm-specific component. Ignoring this intra-

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[6] It is not uncommon for the magnitude of the bias to be large. For example, the biased estimate $\hat{\beta}$ might have the opposite sign from the structural parameter estimate $\hat{\delta}$. Because of this bias, no causal interpretation can be attached to the coefficient estimate of $\hat{\delta}$. However, in the context of prediction, this bias is desirable. It means that the variable is reflecting not only its own impact, but also the impact of other factors omitted from the model. By reflecting some of the effect of these omitted factors, model predictive performance will be enhanced. In dramatic contrast to structural parameter modeling, all else equal, coefficients for variables correlated with omitted factors are informative in the context of prediction.

firm correlation generates biased estimates of the standard errors and renders standard statistical significance tests inappropriate.

Ordinary least squares estimation assumes that the error terms are independent and identically distributed. The independence assumption will be violated in panel data analysis when a firm-specific effect remains in the error term. When both the error term and the independent variable are positively autocorrelated, least squares estimates of the standard errors will be biased in that they understate the true standard errors.

This is the "spurious regression" phenomenon highlighted by, for example, Granger and Newbold (1974) in the time series context. The conventional t-statistic will not have a standard normal limiting distribution, which invalidates the use of, for example, the t-distribution to test the hypothesis that a coefficient is statistically significant. In the presence of autocorrelated series, the number of occasions when $|t|$ is greater than 1.96 is much greater than 5%. The higher the autocorrelation in the series, the greater the probability of observing a t-statistic above $|1.96|$, i.e., the greater the extent to which ordinary least squares standard errors understate the true standard errors. While spurious regression is most widely discussed in a pure time series context, it also comes into play in analysis of panel data. Kao (1999), for example, shows that while it is unrelated to the number of cross-sectional observations, the spurious regression problem increases with the number of time series observations.[7]

To circumvent this problem, we use cluster robust standard errors estimation (White 1984, Arellano 1987), which relaxes the assumption of error independence and allows for correlation within a "cluster," i.e., observations coming for the same firm but in different years. Cluster-robust

[7] Petersen (2007) offers the insight that as the number of time periods of data used in the analysis doubles, ordinary least squares assumes a doubling in the amount of information. However, if the explanatory factors and the error exhibit autocorrelation, the amount of information increases by a factor less than two. Consider the extreme case where both the independent variable and the error are perfectly autocorrelated. Under this scenario, each additional time period provides no additional information and will have no effect on the true standard error. However, the standard errors estimated from ordinary least squares assumes each additional year provides N (the number of cross-sectional observations) additional observations and the estimated standard error will shrink accordingly, albeit incorrectly.

standard errors are a generalization of heteroskedastic robust standard errors (White 1980). By assigning each observation to its own cluster, the approach allows for consistent estimates of standard errors in the presence of correlation of an unknown form within a cluster.

Rather than assuming it to be zero as in least squares analysis, cluster-robust standard errors are based on estimates of the covariance between residuals within a cluster. Under the assumption that the covariance structure is the same across clusters, cluster-robust standard errors provide consistent estimates of the standard errors of the coefficients as the number of clusters grows. The use of robust standard errors does not change the coefficient estimates, but does affect the standard errors and so the t-statistic.[8]

## Data and Measures

We pulled the data from three different sources to create the dataset for our analyses. We obtained brand asset measures from the Young and Rubicam's Brand Asset Valuator (Y&R BAV) database. The stock market data (prices and number of shares) came from the University of Chicago's Center for Research in Security Prices (CRSP) database. We used Standard and Poor's Compustat database to obtain the necessary accounting measures to combine with the stock market data to compute enterprise value and accounting performance measures.

### *Brand Asset Metrics*

Our branding data come from the Y&R's Brand Asset Valuator initiative, which has undertaken large-scale annual surveys of consumer brand perceptions since 2000. We use BAV data

[8] As a sensitivity check, we also estimated a model that allowed for autocorrelated (i.e., dissipating) firm-specific residuals. That is, rather than assuming the same correlational structure for all the firm's observations as depicted by the firm-specific effects model and incorporated in the cluster-robust standard errors, the autocorrelated errors models allows for a decay in the association for years further apart.

In order to test the hypothesis that the coefficient estimate $\hat{\delta}$ is statistically significant in the presence of autocorrelated residuals, we generate a bootstrap distribution that mirrored the procedures undertaken by Granger, Hyung, and Jeon (2001) and Kao (1999). This allowed us to find the 5% critical value for the ratio of the coefficient estimate to the least squares standard errors. Creating a bootstrap distribution is needed because in the presence of autocorrelation the conventional t-statistic does not have a standard normal limiting distribution. By comparing the ratio of the coefficient to the least squares standard errors, we can assess the statistical significance of the estimates. All our conclusions are unaffected by this alternative specification, which is consistent with highly autocorrelated residuals having properties quite similar to firm-specific effects.

that was collected during the 4th quarter of each year for the period 2000-2006. Y&R also undertook brand surveys on a sporadic basis prior to 2000, and we make use of data obtained in the two earlier data collection waves (undertaken in the 1st quarter of 1997 and the 2nd quarter of 1999), for our out-of-sample analysis purposes. Over 2,000 brands are included into each survey wave. Among the surveyed brands we have identified a set of 250 publicly traded mono-brand firms for which complete accounting and stock market data are available for at least some of the 2000-2006 period.

We focus on the five pillars of the Y&R BAV model: perceived brand Differentiation, Relevance, Esteem, Knowledge, and Energy. Differentiation captures perceived distinctiveness of the brand. Relevance measures perceptions of personal relevance, appropriateness, and the importance of the brand. Esteem assesses the level of regard consumers hold for the brand and the valence of consumer attitude. Knowledge is a measure of familiarity and understanding of the brand identity. Energy measures consumer perceptions of brand innovativeness and dynamism. It reflects a brand's ability to meet consumers' future needs and respond to changing conditions.[9]

Raw BAV perceptual brand metrics are collected on different scales, some on a 7-point scale and others as a percent of respondents viewing the brand as possessing a given attribute. To allow for comparability of the coefficients and relative impact of individual brand attributes, we z-standardize each of the measures.

***Measure of Firm Value***

Because our focus is on valuing a business, we use "Enterprise Value" as our measure of firm value. Enterprise value of firm i in time period t is computed as market capitalization of the firm plus its debt, plus minority interest and preferred shares, minus total cash and cash equivalents:

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[9] See, for example, Mizik and Jacobson (2008) for a detailed discussion of the individual metrics and an overview of the 5-pillar BAV model, which adds Energy to the previous four-pillar BAV model (Agres and Dubitsky, 1996). Although the BAV model and results have been widely discussed (e.g., Aaker 1996, Keller 1998), because of its proprietary status for Y&R, BAV data have had only limited use in academic research. Nonetheless, Y&R does grant access to its data and it has been used in some academic research (e.g., Bronnenberg, Dhar, and Dubé 2005; Romaniuk, Sharp, and Ehrenberg 2007; Mizik and Jacobson 2008).

$$\text{Enterprise Value}_{it} = \text{Market Cap}_{it} + \text{Debt}_{it} + \text{Minority Interest}_{it} + \text{Preferred Stock}_{it} - \text{Cash}_{it}.$$

Enterprise value better reflects the cost of buying a company than does market value in that, for example, it takes into account debt (which increases the purchase costs) and cash (which offsets some of the costs). Enterprise value summarizes the claims of all the security holders: debt holders, preferred shareholders, minority shareholders, in addition to the claims of common equity holders. The enterprise value measure is capital structure-neutral, and as such, it is more appropriate to use when analyzing companies that have differing capital structures (Bhojraj and Lee 2002).

***Measures of Accounting Performance***

We used the COMPUSTAT databases to obtain quarterly accounting information that we converted into annual measures. Because some firms in our data sample have fiscal year ends other than December, we utilize the quarterly rather than the annual COMPUSTAT database. For balance sheet items, which we use in computing Enterprise Value, we use the measure at the end of the calendar year. For income statement items, we annualize the measure by taking the sum of the four quarterly values for the calendar year. Our two key measures of accounting performance are sales and return on sales, measured as operating income before depreciation divided by sales.

We have chosen to work with sales (revenues) as the standardizing variable in our valuation application rather than, for example, an earnings or book value-based measures for several reasons. First, sales data are more readily tracked and available for the individual brands or business units of a firm than are other performance metrics. As such, in the absence of available earnings data, our model could still be estimated and utilized with only a slight modification of the estimating equation, i.e., Return on Sales would be omitted as an explanatory factor. Second, earnings and cash flows can be negative and will make valuation impossible or may introduce significant sample selection biases (Liu, Nissim and Thomas 2002). Third, sales are less affected by accounting manipulation. Since investors are now more cautious about relying on accounting earnings data, it is advantageous to make use of measures that are less affected by discretionary accounting choices (Damodaran, 2002).

Finally, marketing managers are more comfortable thinking in terms of sales multiples. However, since our model includes return on sales as a factor explaining the value-to-sales ratio, our analysis incorporates considerations addressed in analysis seeking to control for factors affecting the value-to-earnings multiplier.

***Classifying Sectors***

Sales multiples differ across firms because of industry-wide or sector-specific effects (e.g., growth prospects and risk). As such, understanding what factors influence a sales multiple requires forming sector groupings. A number of different approaches, with both advantages and disadvantages, have been used in past research. Deciding on the number of sectors typically involves a trade-off between homogeneity and a sufficient sample size to provide an accurate estimate of the sector mean. While we undertook sensitivity assessments, for our primary analyses we focused on and allowed for 7 different sectors. These sectors are: (i) Industrial, (ii) Finance, (iii) Retail & Apparel, (iv) High Technology , (v) Consumer Non-durables, (vi) Consumer Durables, and (vii) Travel & Transport. In our analysis we allow for sector-specific annual effects (i.e., the intercept in our models differs by sector for a given year) and for differential effects by sector (i.e., the slope coefficients in our model differ by sector).

***Summary Statistics***

Merging the three data sources resulted in a total of 1,244 pooled cross-sectional time series observations with a complete set of data available for the firm. Table 1 provides summary mean statistics for the measures used in our analysis for the entire sample and by sector. We observe a variety of notable, and expected, differences in the brand metric scores across sectors. For example, the High Tech sector has the highest rating on energy. The Consumer Non-durable sector has the highest ranking on Relevance. The mean of the log(Value-to-Sales) ratio is .317 for the entire sample, which equates to a 1.37 Value-to-Sales multiple.[10] Substantial differences in the ratio exist across sectors. The mean log (Value-to-Sales) ratio is the highest for the Financial sector (.983) and lowest for the Retail and Apparel sector (-.137). It can be quickly observed that these differences are associated with differences in margin. That is, sectors with higher (lower) mean Return on Sales tend to have higher (lower) Value-to-Sales ratios.

***Models***

What remains to be seen, and what we assess in our subsequent empirical analysis, is whether, which, and to what extent do brand metrics provide incremental explanatory power to return on sales in explaining the value-to-sales ratio. We undertake this assessment based primarily on two models. The first model, i.e., Equation 9, uses aggregate analysis and links the log of the enterprise value-to-sales ratio to return on sales, the five Y&R brand metrics, and sector specific annual dummy variables.

$$[9]\ \log(\phi_{it}) = \delta_2 * ROS_{it} + \sum_{b=1}^{5} \beta_b * \text{Brand Asset}_{bit} + \sum_{t=1}^{T} \sum_{k=1}^{K} \gamma(k,t) * S(k,t) + \eta_{it}$$

The second model is similar in format to Equation 9 but allows for sector-specific differences in the slope coefficient. That is, we estimate Equation 10 for each of our 7 sectors separately.

$$[10]\ \log(\phi_{it}) = \delta(k)_2 * ROS_{it} + \sum_{b=1}^{5} \beta(k)_b * \text{Brand Asset}_{bit} + \sum_{t=1}^{T} \gamma(k,t) * Y(t) + \eta_{it},$$

where Y(t) is an indicator function that takes on the value of 1 if the period t; 0 otherwise and all other variables are defined as before.

Estimating equations [9] and [10] allows us to assess to what extent do the brand metrics provide incremental predictive power to return-on-sales in explaining the enterprise value-to-sales ratio.

## Empirical Analysis

[10] We work with logarithms of Value-to-Sales so as to minimize the role of outliers. Other approaches, e.g., winsorizing the data, generate similar findings to those we report.

### ***Brand Impact on Contemporaneous Sales***

The results of estimating equation [4] provide background information as to the impact of brand assets on sales and are presented in Table 2. We find that two brand asset components, Knowledge and Esteem, have significant impact on contemporaneous sales. Among the five brand asset metrics, Knowledge shows the greatest positive impact on sales (.073). This effect is significant at 1% level. Esteem also has a positive impact on sales (.031), which is significant at the 5% level. Differentiation, Relevance, and Energy do not show statistically significant impacts on contemporaneous sales.[11]

Limiting the effect of brand on business value to just their influence on sales overlooks the full effect of brand assets. That is, brand assets may not just affect sales, but also the sales multiplier.

### ***Brand Impact on Valuation Multipliers***

#### ***<u>1. Aggregate Analysis</u>***

We begin our multiplier analysis by regressing the log (Value-to-Sales ratio) on Return on Sales and annual dummy variables for all firms in our data set. Equation 3.1 of Table 3 reports the results, which highlights the positive association between Return on Sales and Value-to-Sales. The estimated coefficient is 5.41 and is statistically significant at the 1% level.[12] As discussed previously, this finding is expected as firms with higher profit per dollar sales create more value per dollar sales than those with lower profit margins. Although not reported in the Table, the equation also includes sector-specific annual dummy variables. The most notable, but also expected, feature of these dummies is that the coefficients for the high-tech sector dummies are higher than in any other sector.

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[11] To assess the possibility that brand effects differ across sectors, we re-estimated model [4] allowing the effects of brand assets to vary by sector. We did not find statistically significant inter-sector differences. The F-statistic (.97) assessing the restriction that the brand effects are the same across the sectors was below the 5% critical value of 1.47.

[12] While the difference in cluster-robust standard errors relative to OLSQ standard errors differs by model, the difference is most notable for the estimated standard error for the return on sales coefficient. The cluster-robust standard errors are approximately twice size of the OLSQ standard errors. The extent of association, however, is extensive enough so that conclusions as to statistical significance are not affected. The increase in the standard errors for the coefficients for the brand metrics is not as dramatic, but as the strength of the association is not as strong, conclusions as to statistical significance are impacted.

This finding is consistent with the high-tech sector firms facing greater growth opportunities during the study period. The $R^2$ statistic indicates that the model is able to explain about 61% of the variation in the data, with return on sales accounting for about 60% of that explanatory power and the annual sector dummies accounting for the remaining 40%.

We expand Equation 3.1 to include the five BAV pillars (i.e., Differentiation, Relevance, Esteem, Knowledge, and Energy). Equation 3.2 of Table 3 reports the results of estimating this model, i.e., equation [9]. We see some evidence of brand effects. Differentiation, Relevance, and Energy have positive effects (.098, .069, and .070, respectively), but only Differentiation is significant at the 5% level, with Energy significant at the 10% level. Knowledge has a negative estimated effect (-.093), and it too is significant at the 10%, but not at the 5% level. The estimated coefficient for Esteem (-.016) is not significantly different from zero.

The negative coefficient for Knowledge should not be interpreted as brand familiarity causing a decrease in firm value. First, as discussed previously, the coefficient estimates cannot be treated as structural parameters. The intent of the analysis is for the coefficient estimates to be as reflective as possible of the information contained in the metric, which is central in a forecasting context, rather than in isolating the causal effect of the variable. We can, however, conclude that the Knowledge metric, after controlling for other brand dimensions, is reflective of information that is negatively correlated with the value-to-sales ratio. So, for instance, the Knowledge metric may have a positive correlation with the maturity of the firm. The maturity of the firm is likely to be negatively correlated with future growth prospects and so the value-to-sales ratio. Therefore, the negative coefficient for Knowledge may be reflecting some of this effect. Second, as we reported in Table 2, Knowledge has a significant positive effect on sales. Because the sales metric sales is in the denominator in the Value-to-Sales multiplier, this result indicates that the bulk of the performance effect of Knowledge is associated with contemporaneous sales (the denominator), rather than with the forward-looking value measure, which is in the numerator of the multiplier metric. Conversely, the positive coefficient for Differentiation indicates that is has a greater association with the future-term affects reflected in the value numerator, rather than the sales denominator.

While Equation 3.1 shows some statistically significant brand effects, the incremental explanatory power gained by adding brand variables to the model is modest. The $R^2$ statistic in Equation 3.2 (.6335) is only 3.6% greater than the $R^2$ statistic in Equation 3.1 (.6115). Another indication of limited incremental increase in explanatory power gained by adding brand metrics to the model comes from an assessment of mean absolute error (MAE), i.e., the average of the absolute value of the error terms. The MAE is .391 from Equation 3.1; it is .382 in Equation 3.2. This is an improvement of only 2.4%. The brand variables in Equation 3.2 provide only a minor increase in predictive power over Equation 3.1, which does not take into account explicit brand measures.

A number of possible considerations can account for this limited increase in explanatory power. One possibility is that brand effects are already reflected in current-term accounting metrics. That is, the financial market's anticipation of brand effects reflected in the Enterprise Value is similarly captured in current period sales and in current-term return on sales. That is, brand variables impact current-period sales and current period Return on Sales. The brand effects showing up in Equation 3.2 reflect only the effects incremental to those running through current-term accounting measures.

Another possibility is that brand effects differ across sectors. While the models in Table 3 include sector-specific annual dummy variables, which allow for differential sector effects to influence the intercept, the specification does not allow for the effect of profitability and brand (i.e., the slope coefficients) to differ by sector. In order to assess this possibility of differential brand effects by sector, we estimate separate models by sector. If the effects of Return on Sales and brand assets differ by sector, aggregation across sectors could result in a masking of the role of brand in influencing the Value-to-Sales ratio. Alternatively, as the effect of Return on Sales is also allowed to differ by sector, these sector-specific models could show that brand metrics have an even smaller effect than those reported in Table 3.

## ***2. Sector-Specific Analysis***

Table 4 reports the results of the sector-specific analysis. Equations 4.11 through 4.17 of Table 4 Panel A report the estimated coefficients from sector-specific models linking Value-to-Sales to Return on Sales and sector-specific annual dummy variables. Return on Sales has a statistically significant effect for each of the sectors, but the magnitude of the response coefficient differs across sectors. It is largest for Retail & Apparel firms (8.97) and smallest for high technology firms (3.74).

Table 4, Panel B reports the results of linking Value-to-Sales to Return on Sales, the five brand metrics, and sector-specific annual dummy variables, i.e., estimating equation [10]. Equations 4.21 through 4.27 in Table 4 Panel B report the results. Here too we observe significant differences in estimated effects.

### Industrial Firms

For industrial firms, we see evidence of a statistically significant association for Energy, with an estimated coefficient of .148. The coefficients for Differentiation, Relevance, and Esteem are both substantially smaller and insignificant. Knowledge has a similar estimated effect to Energy (.155), but it is not statistically significant. In terms of MAE, Equation 4.21 provides a 17.4% improvement in predictive power over Equation 4.11

### Financial Firms, Durable Goods Firms, and Travel & Transport Firms

Three sectors, namely, Financial, Non-Durable Goods, and Travel & Transport, have very similar estimated models. The estimated coefficients for return on sales in these sectors are very similar, i.e., 4.81, 4.67, and 4.95, respectively. Further, for each of these sectors, the only brand metric having a statistically significant effect is Differentiation. Here too the estimated effects are quite similar (.472, .304, and .382, respectively). In terms of MAE, brand metrics provide the greatest improvement in predictive power for Durable Goods firms (30%), with a 16% improvement for Travel & Transport firms and a 7.6% improvement for Financial firms.

Retail & Apparel Firms

For Retail & Apparel firms, Return on Sales has a very large estimated effect (8.83). None of the brand effects are significant individually and the joint hypothesis that all 5 of the brand coefficients are zero cannot be rejected. Consistent with the lack of effect, Equation 4.23 provides only a 2% decrease in MAE compared to Equation 4.13. To further verify this finding, we undertook additional analyses where we separate this sector into apparel firms and retail firms. We find results consistent with the aggregate analysis. Firms in both these sub-sectors share the common feature that their value-to-sales multiplier is not affected by the brand attributes and is highly responsive to Return on Sales. Indeed, the coefficient on ROS for these firms is the highest among the sectors we examined.

High Technology Firms

Consistent with an accounting literature that notes that valuation models perform worst in dynamic industries, we observe the lowest $R^2$ statistic and the highest mean absolute error in the High Technology sector. The impact of Return of Sales is also the smallest (3.98) for High Technology firms. Current profitability is less informative about future performance than in other sectors. We find statistically significant effects for three brand variables, i.e., Differentiation, Relevance, and Knowledge. While Differentiation and Relevance have positive effects (.185 and .416), the estimated effect of Knowledge is negative (-.287). Equation 4.24 provides a 7% decrease in MAE compared to Equation 4.14.

Consumer Non-Durables

For Consumer Non-Durables, the return on sales effect is of a relatively large magnitude (7.44). Knowledge is the only brand asset having a statistically significant effect (.191). The other brand metrics are statistically insignificant and the joint hypothesis that these 4 brand metrics as a group are equal to zero cannot be rejected. Equation 4.25 shows an improvement in MAE of 9% over Equation 4.15.

Summary

Analysis based on sector-specific value multiple models provides different insights from those implied by the aggregate model. While the aggregate analysis shows statistically significant brand effects providing little improvement in predictive power (i.e., only a 2.4% improvement in MAE), the sector-specific models suggest that the brand metrics provide substantially more predictive power. Only for the Retail & Apparel sector do we find as limited impact of brand variables as we do in the aggregate analysis. For the other six sectors, we are seeing gains in predictive power ranging from 7% to 30%, with the average across these six sectors being 10%.

## Sensitivity Analysis

To assess the stability and validity of our results we undertook several additional analyses. In particular, we examined the performance of our valuation model out-of-sample to determine whether conclusions drawn from Table 4 extend to other time periods. We also undertook a factor analysis to assess whether a more parsimonious grouping of brand attributes would be more appropriate for valuation purposes. We also assessed the valuation framework by examining model predictive performance in the absence of profit margin data.

*Out-of-Sample Predictive Accuracy*

While assessments of statistical significance and within-sample predictive accuracy improvement have merit, an additional model assessment tool is the extent to which the model parameters can be used out-of-sample to predict the value-to-sales ratio. Because, for example, model parameters may not be estimated with sufficient accuracy or model parameters may change over time, differences may exist between in-sample performance and out-of-sample performance. Indeed, a more comprehensive model that offers superior explanatory power in-sample may yield inferior predictions to a more parsimonious model out-of-sample. As such, we sought to assess the ability of Equation 4.2 and the model parameters reported in Table 4 to predict value-to-sales ratios for periods other than for the periods 2000-2006, i.e., the period for which the model parameters were estimated.

Y&R also engaged in brand surveys in 1997 and 1999. We did not include these waves of data in our analysis because their sampling was at unequal intervals from the other survey waves that we used in our analysis, which would have an impact on some of our analysis, in particular, our sensitivity analysis based on an autoregressive error structure. Further, excluding these years of data from the model estimation provided us a means to undertake an out-of-sample assessment of predictive accuracy. Using the parameter estimates from Table 4, we assess how much predictive accuracy is gained by incorporating brand metrics into a value-to-sales multiplier model.

Table 5 reports the MAE across the six sectors in Table 4 showing brand effects. We excluded Retail & Apparel firms from this analysis as Equation 4.4 showed no role for brand effects in-sample (i.e., the estimated brand coefficients were both small and statistically insignificant). As such, we had no reason to believe they would provide any improvement in predictive power out-of-sample. And, in fact, this is confirmed empirically. Table 5 shows estimates from four predictive models. Model 5.1 uses the annual sector mean of the value-to-sales ratio as the prediction. Model 5.2 makes use of the annual sector mean and the Return on Sales for the firm multiplied by the estimated coefficients in Table 4, Panel A to predict the value-to-sales multiplier. Model 5.3 extends Model 5.2 by including brand metrics and the estimated coefficients in Table 4, Panel B. Model 5.4 excludes Return on Sales and bases predictions on the annual sector mean and the brand metrics.

Model 5.1 has a MAE of .57. Taking into account Return on Sales reduces the MAE to .3483, i.e., a 64% improvement. Including brand metrics into the analysis further enhances predictive power as the MAE is .2993. This represents a 16% reduction in prediction error. As such, the out-of-sample improvement predictive power is actually greater than the in-sample results (which showed a 10%

reduction in MAE). Interestingly, predictions made for firms both for 1997 and for 1999 show a very similar improvement of 16% each.

Lack of Available Earnings Data

In some instances, earnings data are not available for use in a valuation analysis. This can occur, for example, because of the proprietary nature of earnings information. While sales typically can be obtained from outside sources, obtaining earnings data for a firm typically requires the cooperation of the firm’s managers. This cooperation may not be forthcoming. In such a case, valuation can proceed in the absence of earnings information with a simple modification of Model 4.2. Namely, Return on Sales as an explanatory factor can be dropped from the estimating equation. This modified model links the value-to-sales ratio to annual sector dummies and brand metrics. To the extent that brand metrics are correlated with current-period Return on Sales, their estimated coefficients will reflect some of the effect of Return on Sales when it is excluded from the model.

We undertook analysis with this modified model and assessed the out-of-sample predictive power of the model. As reported in Model 5.4 of Table 5, the model generated an out-of-sample MAE of .4366. This represents a 31% improvement in predictive accuracy over a model based just on annual sector means. It is, however, a 26% reduction in accuracy compared to predictions based on the sector mean and Return on Sales. This reduction was expected given that not all of the variation in firms’ return on sales can be explained by brand attributes. However, the improvement in predictive accuracy over the sector mean further supports the use of brand metrics for valuation purposes.

*Factor Analysis Assessment*

As the brand metrics are correlated with each other, collinearity may induce inaccuracies in the coefficient estimates. While collinearity still allows for unbiased coefficient estimates, they may not be estimated with sufficient precision (as evidenced by larger standard errors). One approach for dealing with this potential issue would involve undertaking factor analysis on the brand metrics to reduce their dimensionality and then relating the resulting factors to the value-to-sales ratio.

We undertook this analysis and obtained a two-factor solution. One factor was "Relevant Stature" (i.e., a factor roughly of an equal weighting of Relevance, Knowledge, and Esteem) and the other was "Differentiated Energy" (i.e., a factor primarily based on Differentiation and Energy). We then replaced the five brand metrics in Equation 4.2 with these two factors. While we found statistically significant brand effects using these two brand factors, the explanatory power exhibited by the brand factors was noticeably diminished compared to the models allowing for separate brand effects. While Relevance and Knowledge both load positively on one factor, they tend to exhibit differing associations with the value-to-sales ratio. This observation is most prominent for the High Tech sector (i.e., Equation 4.24) where Relevance has a positive effect, while Knowledge has a negative effect. While factor analysis essentially makes use of the average of Relevance and Knowledge, Equation 4.24 results suggest that the difference between Relevance and Knowledge would be a more appropriate measure.

While aggregation will always result in a loss of explanatory power within sample, out-of-sample analysis sometimes may yield different conclusions. This, however, is not the case in this instance. We found that the 2-factor brand model had an out-of-sample MAE of .337. This represents only a 3% decrease in MAE from the analysis based just on sector effects and return on sales data and is a 13% increase in MAE over analysis allowing for separate effects for each of the five brand metrics. As such, the out-of-sample analysis is fully supportive of the in-sample analysis in indicating that a factor analysis-based approach is inferior to allowing for separate brand effects.

## Conclusion

We have proposed a framework to incorporate brand assets into a relative businesses valuation approach. We have demonstrated its use and performance using Y&R BAV brand metrics.

While our analyses show that the brand asset, through the dimensions Knowledge and Esteem, impact sales, brands also have a role in influencing firm valuation through the sales multiple.

We find that brand metrics provide significant incremental explanatory power to profitability measures in explaining the Value-to-Sales ratio. Incorporating brand-related information into the valuation model enhances out-of-sample predictive power by 16%. The importance and the impact of brand dimensions, however, are different across sectors. For the seven sectors we examine, brand metrics reduce forecast errors from 2.4% in retail and apparel to 30% in consumer durable goods.

The goal of our analysis focused on assessing whether brand metrics enhanced predictions of the value-to-sales ratio. In this forecasting context, rather than seeking to obtain unbiased coefficient estimates of the causal effect, the goal is to extract as much information from the brand metrics as possible. As such, the estimated parameters for the brand metrics reflect not just their causal impact on the value-to-sales ratio but also their ability to depict the effect of other factors omitted from the model. In contrast to least squares standard errors that tend to understate confidence intervals, the use of cluster-robust standard errors allows for assessing statistical significance in the presence of omitted firm-specific variables. However, the coefficient estimates cannot be interpreted as structural parameters. This makes it problematic to attach causal interpretations to the estimated model coefficients.

However, we can say that brand metrics have different association with the value-to-sales ratio across different industrial sectors. For example, we find that Differentiation is the brand metric most reflective of information influencing sales multiples in a number of sectors. A possible explanation for this finding is that as product markets and services are becoming more and more commoditized and the offerings are similarly priced, brand Differentiation is a key factor in attracting consumers. Differentiation is needed to stand out in the crowded space of similar offerings.

Interestingly, we find no association between the brand metrics and value-to-sales ratio in our retail and apparel sector. In this sector, profit margin is the only significant predictor of the value-to-sales multiplier. Perhaps, consumer tastes and preferences for apparel change too fast with the seasonal fashion, so that current brand attitudes and perceptions in this sector have little predictive value incremental to profit margin about future financial performance. That is, if brand dimensions are not associated with information impacting current-term performance (sales or profit margin), they are not viewed by the financial markets as likely to impact future-term performance.

We see several promising avenues and future research opportunities in this area. First, we have focused on five key metrics comprising the BAV model (Differentiation, Relevance, Esteem, Knowledge, and Energy). It is possible that some other brand asset component might add incremental explanatory power and improve valuation predictive performance in some or all sectors. As part of additional sensitivity analyses, we found that the Y&R brand metric “gaining in popularity” was related to the Value-to-Sales ratio for firms in the Retail and Apparel sector. However, while statistically significant, the metric provided only very modest improvements in explanatory power both in-sample and out-of-sample. Still, the possibility that other brand metrics might provide additional explanatory power should be further explored.

Further, other intangible assets, such as management quality, customer satisfaction, firm’s innovation strategy, measures of technological capabilities such as patents and patent citations, and product pipeline, might also play a role in influencing firm valuation and, as such, might provide improvement in forecasting business valuation. If consistent measures of other intangible assets are available, they can be easily incorporated into our valuation framework. We view this direction as a particularly valuable avenue for future research.

**Table 1**
**Sample Characteristics**
**Time Period: 2000-2006**

The table reports mean values of each data item.

| | **Full Sample** | **Industrial Firms** | **Financial Firms** | **Retail & Apparel Firms** | **High Tech Firms** | **Non-Durable Goods Firms** | **Durable Goods Firms** | **Travel & Transport Firms** |
|---|---|---|---|---|---|---|---|---|
| Log(Value/Sales) | .317 | .208 | .983 | -.137 | .798 | .521 | -.065 | .105 |
| Return on Sales | .171 | .152 | .325 | .121 | .201 | .187 | .137 | .143 |
| Differentiation | .034 | -.324 | -.638 | .501 | -.184 | .103 | .238 | -.386 |
| Relevance | .106 | -.295 | -.821 | .216 | -.268 | 1.138 | .394 | .028 |
| Esteem | .043 | .051 | -.738 | .016 | -.352 | .832 | .928 | .039 |
| Knowledge | -.015 | -.450 | -.342 | .122 | -.609 | .763 | .396 | .439 |
| Energy | .027 | .432 | -.685 | -.244 | .652 | -.401 | .554 | -.261 |
| # observations | 1244 | 128 | 80 | 395 | 304 | 181 | 57 | 99 |

**Variable Definitions for firm i, year t:**

Market Capitalization$_{it}$=price$_{it}$ * shares$_{it}$

Enterprise Value$_{it}$= Market Cap$_{it}$ + Debt$_{it}$ + Minority Interest$_{it}$ + Preferred Stock$_{it}$ - Cash$_{it}$
=price*shares + (data51 + data45) + data53 + data55 - data36)

Operating Income$_{it}$= $\sum_{q=1}^{4} \text{Operating Income before Depreciation}_{iq} = \sum_{q=1}^{4} \text{data21}_{iq}$ , where q is quarter in year t

Sales$_{it}$= $\sum_{q=1}^{4} \text{Sales}_{iq} = \sum_{q=1}^{4} \text{data2}_{iq}$ , where q is quarter in year t;

Enterprise Value-to-Sales$_{it}$ = Enterprise Value$_{it}$/Sales$_{it}$

Return on Sales$_{it}$= Operating Income$_{it}$ /Sales$_{it}$

Brand variables are z-standardized

**Table 2**
**Brand Impact on Contemporaneous Sales**
**Dependent Variable: Sales Growth**
**(N=943)**

**Model:** $\Delta\log(Sales_{it}) = \sum_{b=1}^{5} \theta_b * \Delta Brand\ Asset_{bit} + \sum_{t=1}^{T}\sum_{k=1}^{K} \zeta(k,t) * \Delta S(kt) + \eta_{it}$

| | estimate | t-stat |
|---|---|---|
| Differentiation | -0.00433 | [-0.61] |
| Relevance | 0.00619 | [0.39] |
| Esteem | 0.03123* | [2.20] |
| Knowledge | 0.07259** | [3.11] |
| Energy | -0.00760 | [-1.02] |

Each equation also includes annual sector-specific dummy variables (not reported). t-statistics in brackets.
**significant at the 1% level, *significant at the 5% level

**Table 3**
**Valuation of Branded Businesses: Aggregate Analysis**
**Dependent Variable: Log (Enterprise Value/Sales)**
**(N=1,244)**

**Model 3.1:** $\phi_{it} = \delta_1 * ROS_{it} + \sum_{t=1}^{T}\sum_{k=1}^{K} \gamma(k,t) * S(k,t) + \eta_{it}$

**Model 3.2:** $\phi_{it} = \delta_2 * ROS_{it} + \sum_{b=1}^{5} \beta_b * Brand\ Asset_{bit} + \sum_{t=1}^{T}\sum_{k=1}^{K} \gamma(k,t) * S(k,t) + \eta_{it}$

| | Equation 3.1 | | Equation 3.2 | |
|---|---|---|---|---|
| | estimate | t-stat | estimate | t-stat |
| Return on Sales | 5.41** | [14.16] | 5.33** | [14.98] |
| Differentiation | | | .098* | [2.44] |
| Relevance | | | .069 | [1.17] |
| Esteem | | | -.016 | [-0.27] |
| Knowledge | | | -.093 | [-1.77] |
| Energy | | | .070 | [1.91] |
| $R^2$ | .6115 | | .6335 | |
| MAE | .391 | | .382 | |

Each equation includes annual sector-specific dummy variables (not reported). t-statistics, which are the ratio of the estimated coefficient to the cluster-robust standard error, are in brackets. **significant at the 1% level, *significant at the 5% level.
MAE is mean absolute error and is defined as the mean of absolute value of the forecast error, i.e.,
$|Enterprise\ Value_{it} - \widehat{Enterprise\ Value_{it}}|$.

**Table 4**
**Valuation of Branded Businesses: Analysis by Sector**
**Dependent Variable: Log (Enterprise Value/Sales)**

**Table 4 Panel A: Model 4.1** $\log(\phi_{it}) = \delta_1 * ROS_{it} + \sum_{t=1}^{T} \sum_{k=1}^{K} \gamma(k,t) * S(kt) + \eta_{it}$

| | Equation 4.11 Industrial Firms | Equation 4.12 Financial Firms | Equation 4.13 Retail & Apparel Firms | Equation 4.14 High Tech Firms | Equation 4.15 Non-Durable Goods Firms | Equation 4.16 Durable Goods Firms | Equation 4.17 Travel & Transport Firms |
|---|---|---|---|---|---|---|---|
| Return on Sales | 6.52** [5.42] | 5.14** [5.83] | 8.97** [15.85] | 3.74** [6.65] | 7.69** [9.64] | 7.82** [5.52] | 6.69** [13.29] |
| #obs | 128 | 80 | 395 | 304 | 181 | 57 | 99 |
| $R^2$ | .6308 | .6648 | .5544 | .3747 | .7503 | .6369 | .7027 |
| MAE | .283 | .340 | .347 | .520 | .224 | .289 | .362 |

**Table 4 Panel B: Model 4.2** $\log(\phi_{it}) = \delta(k)_2 * ROS_{it} + \sum_{b=1}^{5} \beta(k)_b * \text{Brand Asset}_{bit} + \sum_{t=1}^{T} \gamma(k,t) * Y(t) + \eta_{it}$

| | Equation 4.21 Industrial Firms | Equation 4.22 Financial Firms | Equation 4.23 Retail & Apparel Firms | Equation 4.24 High Tech Firms | Equation 4.25 Non-Durable Goods Firms | Equation 4.26 Durable Goods Firms | Equation 4.27 Travel & Transport Firms |
|---|---|---|---|---|---|---|---|
| Return on Sales | 6.22** [7.35] | 4.81** [7.33] | 8.83** [13.66] | 3.98** [7.90] | 7.44** [10.88] | 4.67** [5.95] | 4.95** [8.71] |
| Differentiation | .008 [0.10] | .472* [2.09] | .041 [.63] | 0.185* [1.96] | -0.086 [-1.84] | 0.304* [2.51] | 0.383** [3.00] |
| Relevance | .015 [0.16] | .291 [.99] | .084 [.83] | 0.416* [2.27] | -0.044 [-0.62] | 0.251 [1.48] | -0.289 [-1.87] |
| Esteem | -.037 [-0.36] | -.029 [-0.14] | .055 [.60] | -0.207 [-1.23] | -0.025 [-0.42] | -0.175 [-1.69] | 0.032 [0.17] |
| Knowledge | .155 [1.07] | -.053 [-.26] | -.061 [-.69] | -0.287** [-2.69] | 0.191** [2.92] | -0.226 [-1.93] | 0.081 [0.35] |
| Energy | .148** [3.32] | .110 [.61] | -.026 [-0.39] | 0.098 [1.43] | 0.131 [1.75] | 0.093 [0.71] | 0.066 [0.52] |
| #obs | 128 | 80 | 395 | 304 | 181 | 57 | 99 |
| $R^2$ | .7215 | .7391 | .5645 | .4711 | .7772 | .8021 | .7769 |
| MAE | .241 | .316 | .339 | .486 | .205 | .222 | .313 |

Each equation Model 4.1 and 4.2 also includes annual dummy variables. t-statistics, which are the ratio of the estimated coefficient to the cluster-robust standard error, are in brackets. **significant at the 1% level, *significant at the 5% level

MAE is mean absolute error and is defined as the mean of absolute value of the forecast error, i.e., $|\text{Enterprise Value}_{it} - \widehat{\text{Enterprise Value}_{it}}|$.

**Table 5**
**Out-of-Sample Predictions for Years 1997 and 1999**
**Mean Absolute Forecast Error (N=161)***

**Forecasting Models:**

**5.1:** $\widehat{\text{Enterprise Value}_{it}} = \textbf{Sales}_{it} * [\sum_{t=1}^{T}\sum_{k=1}^{k}\hat{\gamma}(\text{k,t}) * \text{S(k,t)}];$

**5.2:** $\widehat{\text{Enterprise Value}_{it}} = \textbf{Sales}_{it} * [\hat{\delta}(k)_1 * \textbf{ROS}_{it} + \sum_{t=1}^{T}\sum_{k=1}^{k}\hat{\gamma}(\text{k,t}) * \text{S(k,t)}];$

**5.3:** $\widehat{\text{Enterprise Value}_{it}} = \textbf{Sales}_{it} * [\hat{\delta}(k)_2 * \textbf{ROS}_{it} + \sum_{b=1}^{5}\hat{\beta}(k)_b * \text{Brand Asset}_{bt} + \sum_{t=1}^{T}\sum_{k=1}^{k}\hat{\gamma}(\text{k,t}) * \text{S(k,t)}]$

**5.4:** $\widehat{\text{Enterprise Value}_{it}} = \textbf{Sales}_{it} * [\sum_{b=1}^{5}\hat{\beta}(k)_b^{'} * \text{Brand Asset}_{bt} + \sum_{t=1}^{T}\sum_{k=1}^{k}\hat{\gamma}(\text{k,t}) * \text{S(k,t)}]$

| Model | Explanatory Variables | MAE | Improvement |
|---|---|---|---|
| 5.1 | Sector Yearly Mean | .5719 | |
| 5.2 | Sector Yearly Mean, Return on Sales | .3483 | 5.2 improvement over 5.1 = 64% |
| 5.3 | Sector Yearly Mean, Return on Sales, BAV Brand Metrics | .2993 | 5.3 improvement over 5.2 = 16% |
| 5.4 | Sector Yearly Mean, BAV Brand Metrics | .4366 | 5.4 improvement over 5.1 = 31% |
| | | | 5.4 reduction compared to 5.2 = 25% |

MAE is mean absolute error and is defined as the mean of absolute value of the prediction error, i.e., $|\text{Enterprise Value}_{it} - \widehat{\text{Enterprise Value}_{it}}|$.

* Retail & Apparel Sector firms excluded from the analysis due to the lack of observed brand impact in-sample.

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